Chapter 6 ESTIMATION



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Learning Outcomes

- > After studying this chapter, the student will:
- 1. understand the importance and basic principles of estimation.
- 2. be able to calculate interval estimates for a variety of parameters.
- 3. be able to interpret a confidence interval from both a practical and a probabilistic viewpoint.

6.1. INTRODUCTION

Fundamental Concepts

Statistical inference is the procedure by which we reach a conclusion about a parameter on the basis of the information contained in a sample drawn from that population.

> A point estimate is a single numerical value used to estimate the corresponding population parameter.

> An interval estimate consists of two numerical values defining a range of values that, with a <u>specified degree of confidence</u>, <u>most likely</u> includes the parameter being estimated.

Interval Estimate Components

> An interval estimate may be expressed as follows:

estimator \pm (reliability coefficient) imes (standard error of estimator)

> Confidence level is the percentage of confidence that the population parameter is between the limits of the interval estimator. It is often denoted by $(1 - \alpha) \times 100\%$.

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Practical Interpretation of Confidence Intervals

> When sampling from the population, we are $(1 - \alpha) \times 100\%$ confident that the single computed interval,

estimator \pm (reliability coefficient) imes (standard error of estimator)

contains the population parameter.

6.2 CONFIDENCE INTERVAL FOR A POPULATION MEAN



FIGURE 6.3.3 Flowchart for use in deciding between *z* and *t* when making inferences about population means. (*Use a nonparametric procedure. See Chapter 13.)



FIGURE 6.3.3 Flowchart for use in deciding between *z* and *t* when making inferences about population means. (*Use a nonparametric procedure. See Chapter 13.)

(1-lpha) imes 100% Confidence Interval for the Mean

Population Variance is known	Population Variance is unknown and sample size is large $(n\geq 30)$
$\overline{x} \pm z_{(1-lpha/2)} \frac{\sigma}{\sqrt{n}}$	$\overline{x} \pm z_{(1-\alpha/2)} \frac{s}{\sqrt{n}}$

such that \bar{x} is the sample mean, $\sigma(s)$ is the population (sample) standard deviation, n is the sample size.

(1-lpha) imes 100% Confidence Interval for the Mean (cont.)

> The reliability coefficient $Z_{(1-\alpha/2)}$ is as follows:

(1-lpha) imes 100%	$z_{(1-\alpha/2)} \approx$
90%	1.64
95%	1.96
99%	2.58

> See Example 6.2.1 to Example 6.2.3.

MegaStat Application

Example 6.2.1

> Suppose a researcher, interested in obtaining an estimate of the average level of some enzyme in a certain human population, takes a sample of 10 individuals, determines the level of the enzyme in each, and computes a sample mean of $\bar{x} = 22$. Suppose further it is known that the variable of interest is approximately normally distributed with a variance of 45. Find the interval estimation for the population mean μ .

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Interval Estimation for μ (MegaStat Application)

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Х Confidence Intervals / Sample Size π Confidence interval: mean Confidence interval: p <u>0</u>K Sample size: p Sample size: mean Sample size: (alpha, <u>b</u>eta) Clear Confidence Level Ca<u>n</u>cel 22 Mean ⊙z⊙t 95% -_ SQRT(45) <u>H</u>elp Std. Dev. -10 n _ lower upper 26.158 17.842 Preview 7. Click Preview to get the confidence interval limits. © FAROUQ MOHAMMAD A. ALAM 17

6.5 CONFIDENCE INTERVAL FOR A POPULATION PROPORTION

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$(1 - \alpha) \times 100\%$ Confidence Interval for the Proportion (using the normal approximation to the binomial distribution)

$$\widehat{p} \pm z_{(1-lpha/2)} \sqrt{rac{\widehat{p}(1-\widehat{p})}{n}}$$

such that \hat{p} is the sample proportion, n is the sample size and the reliability coefficient $Z_{(1-\alpha/2)}$ is as follows:

(1-lpha) imes 100%	$z_{(1-\alpha/2)} \approx$
90%	1.64
95%	1.96
99%	2.58

MegaStat Application

Example 6.5.1.

> A report published in 2003 reported that 18 percent of Internet users have used it to search for information regarding experimental treatments or medicines. The sample consisted of 1220 adult Internet users. Construct a 95% confidence interval for the proportion of Internet users in the sampled population who have searched for information on experimental treatments or medicines.

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> A report published in 2003 reported that 18 percent of Internet users have used it to search for information regarding experimental treatments or medicines. The sample consisted of 1220 adult Internet users. Construct a 95% confidence interval for the proportion of Internet users in the sampled population who have searched for information on experimental treatments or medicines.

Interval Estimation for p (MegaStat Application)

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4. Add the value of sample proportion.

 \mathcal{T}

3. Choose the tab "Confidence interval: p".

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