Chapter 6

ESTIMATION

Learning Outcomes

- **After studying this chapter, the student will:**

1. understand the importance and basic principles of estimation.
2. be able to calculate interval estimates for a variety of parameters.
3. be able to interpret a confidence interval from both a practical and a probabilistic viewpoint.

6.1. INTRODUCTION

Fundamental Concepts

- **Statistical inference** is the **procedure** by which we **reach a conclusion** about a **parameter** on the **basis** of the **information** contained in a **sample** drawn from that **population**.

- A **point estimate** is a **single numerical value** used to **estimate** the corresponding **population** parameter.

- An **interval estimate** consists of **two numerical values** defining a **range of values** that, with a **specified degree of confidence**, **most likely includes the parameter** being **estimated**.

Interval Estimate Components

- An interval estimate may be expressed as follows:

estimator $\pm$ (reliability coefficient) $\times$ (standard error of estimator)

- **Confidence level** is the **percentage of confidence** that the population **parameter is between the limits of the interval estimator**. It is often denoted by $(1-\alpha)\times 100\%$.

Practical Interpretation of Confidence Intervals

› When sampling from the population, we are $(1-\alpha)\times 100\%$ confident that the single computed interval,

estimator $\pm$ (reliability coefficient) $\times$ (standard error of estimator)

contains the population parameter.

6.2 CONFIDENCE INTERVAL FOR A POPULATION MEAN

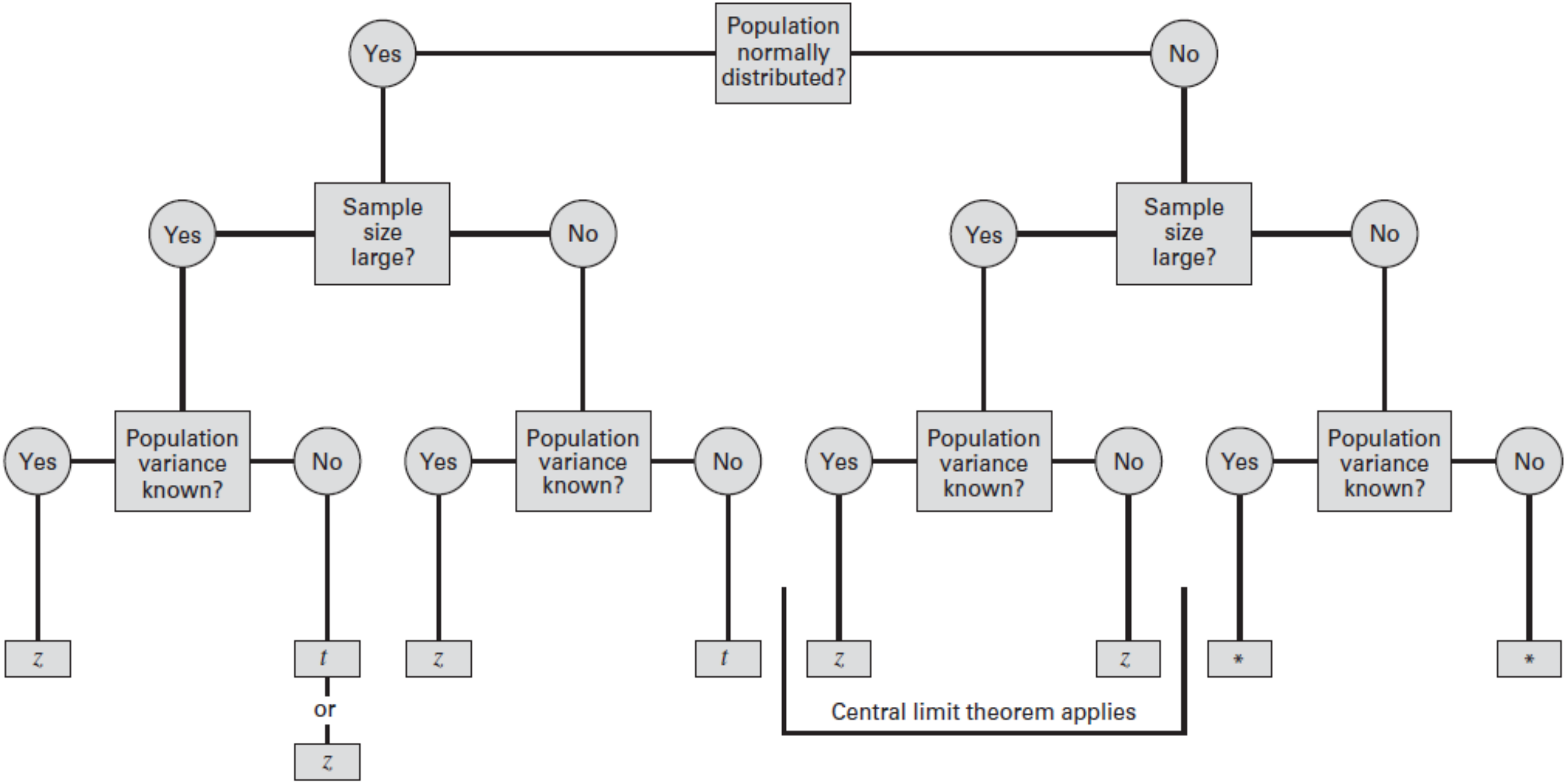

FIGURE 6.3.3 Flowchart for use in deciding between z and t when making inferences about population means. (*Use a nonparametric procedure. See Chapter 13.)

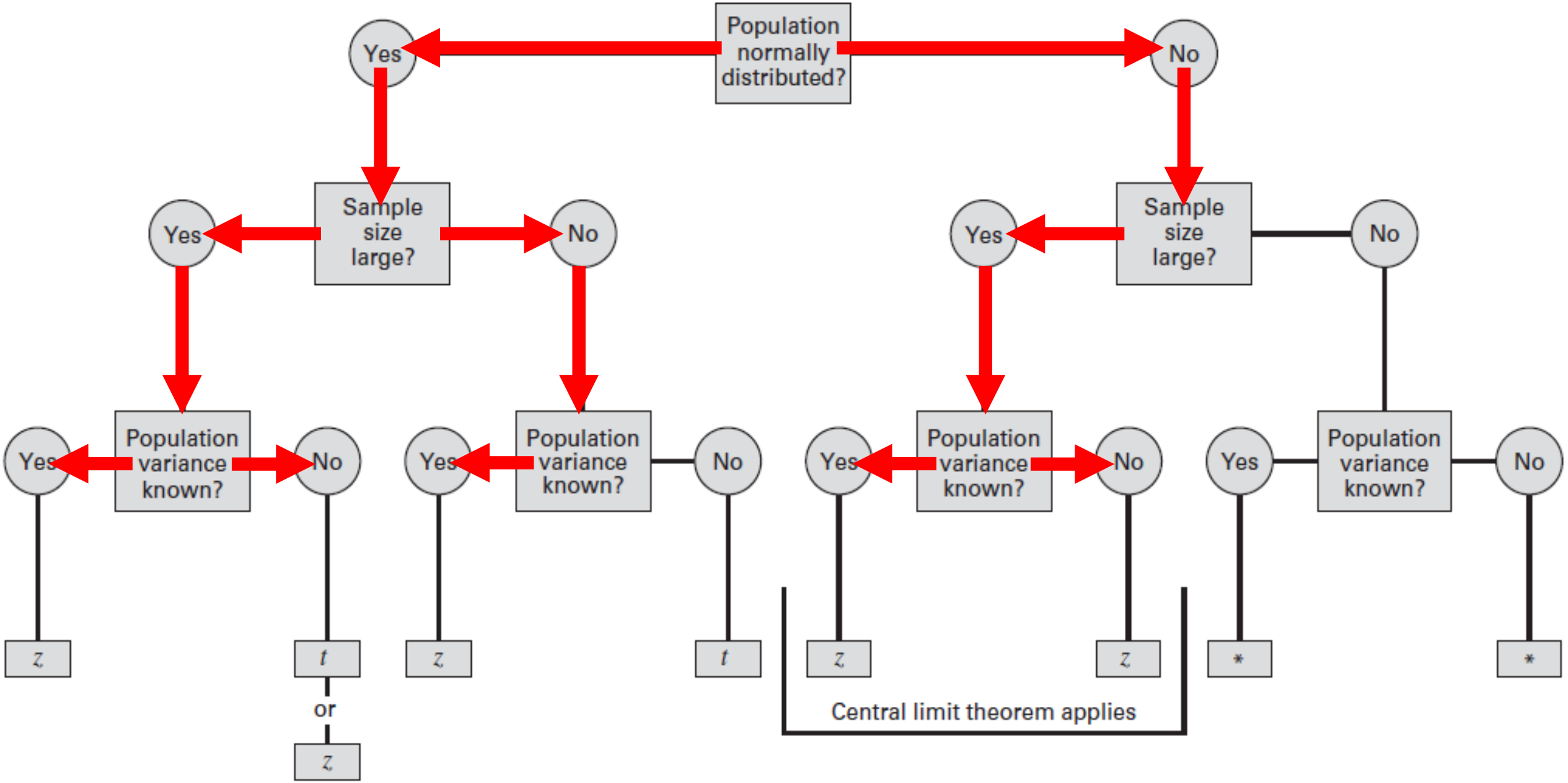

FIGURE 6.3.3 Flowchart for use in deciding between z and t when making inferences about population means. (*Use a nonparametric procedure. See Chapter 13.)

$(1-\alpha)\times 100\%$ Confidence Interval for the Mean

Population Variance is known	Population Variance is unknown and sample size is large $(n \geq 30)$
$\bar{x} \pm z_{(1-\alpha/2)} \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm z_{(1-\alpha/2)} \frac{s}{\sqrt{n}}$

such that $\bar{x}$ is the **sample mean**, $\sigma(s)$ is the **population (sample) standard deviation**, n is the **sample size**.

$(1-\alpha)\times 100\%$ Confidence Interval for the Mean (cont.)

- The reliability coefficient $z_{(1-\alpha/2)}$ is as follows:

$(1-\alpha)\times 100\%$	$z_{(1-\alpha/2)} \approx$
90%	1.64
95%	1.96
99%	2.58

- See **Example 6.2.1** to **Example 6.2.3**.

MegaStat Application

Example 6.2.1

- Suppose a researcher, interested in obtaining an estimate of the average level of some enzyme in a certain human population, takes a sample of 10 individuals, determines the level of the enzyme in each, and computes a sample mean of $\bar{x} = 22$. Suppose further it is known that the variable of interest is approximately normally distributed with a variance of 45. Find the interval estimation for the population mean μ.

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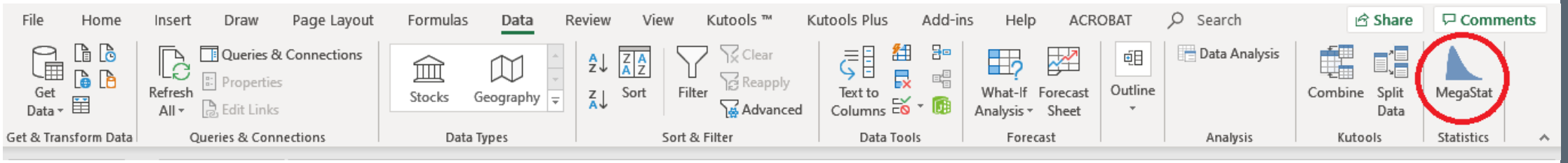
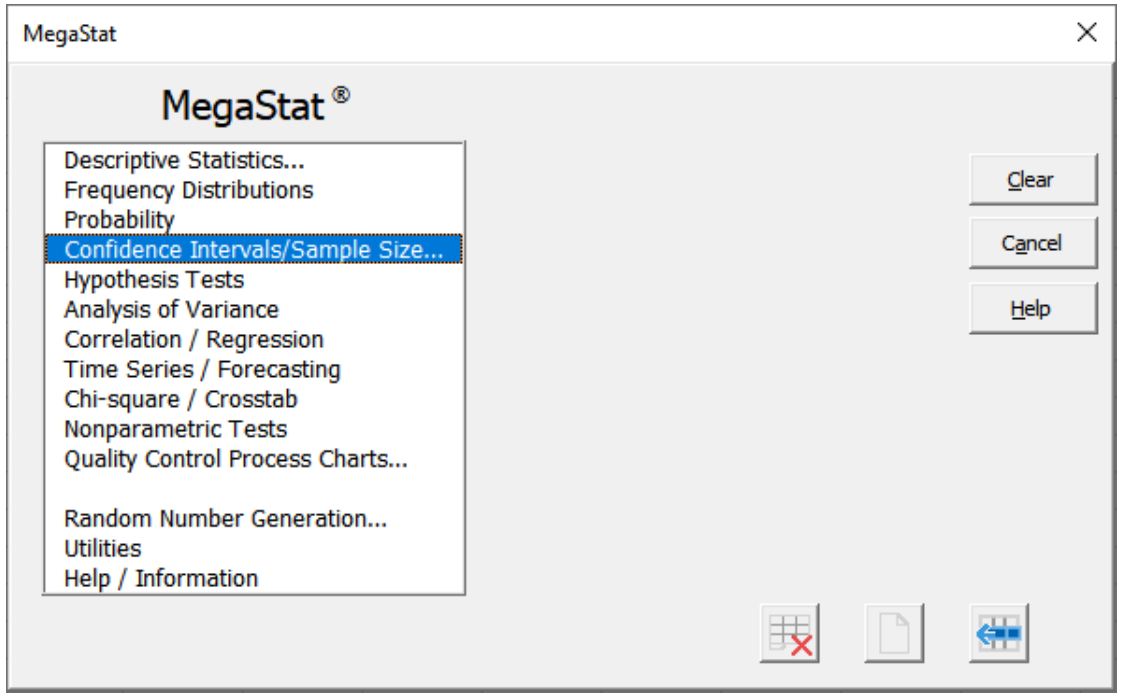
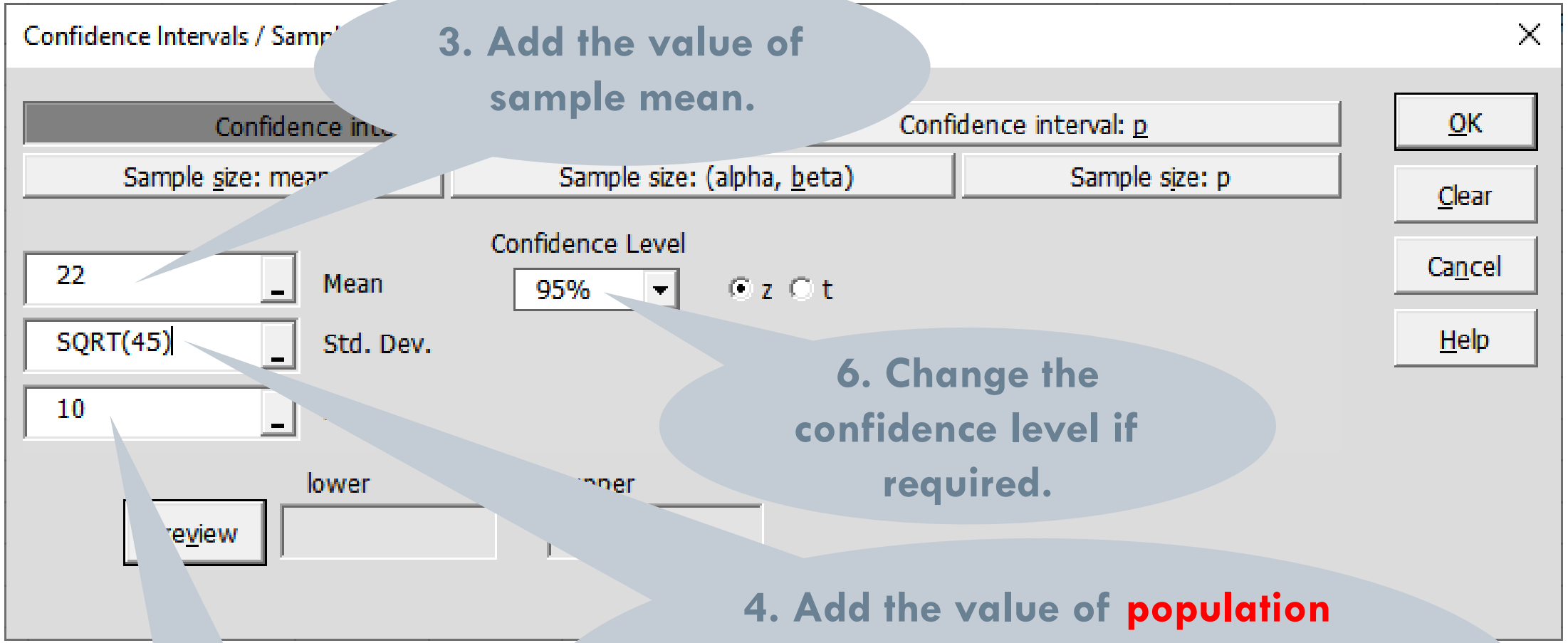
Interval Estimation for μ (MegaStat Application)





1. In **Data Ribbon**, Click on MegaStat icon.
2. Select **Confidence Intervals/Sample Size**

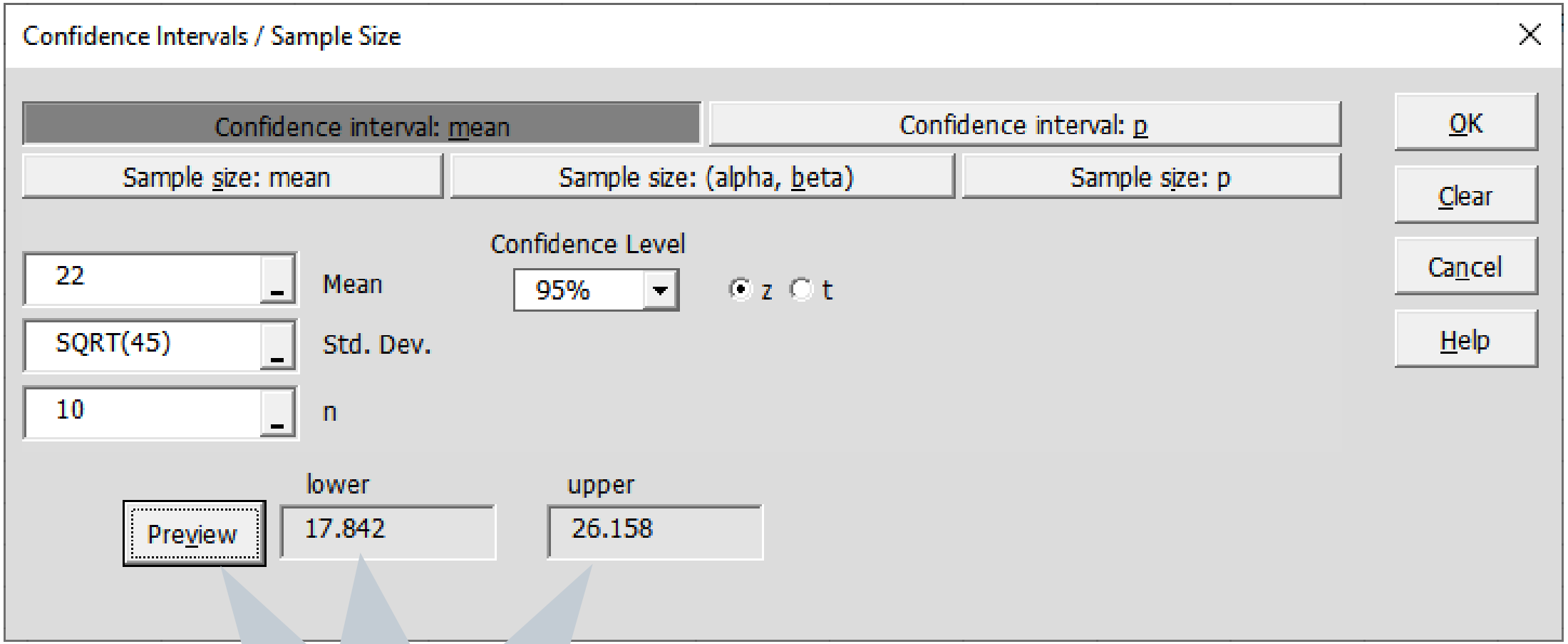


3. Add the value of sample mean.

6. Change the confidence level if required.

4. Add the value of **population standard deviation OR** the **sample standard deviation if $n \geq 30$**. Use **SQRT(45)** to get the **standard deviation from the variance.**

5. Add the value of the sample size.



7. Click Preview to get the confidence interval limits.

6.5 CONFIDENCE INTERVAL FOR A POPULATION PROPORTION

$(1-\alpha)\times 100\%$ Confidence Interval for the Proportion
(using the normal approximation to the binomial distribution)

$$\hat{p} \pm z_{(1-\alpha/2)}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

such that $\hat{p}$ is the sample proportion, n is the sample size and the reliability coefficient $z_{(1-\alpha/2)}$ is as follows:

$(1-\alpha)\times 100\%$	$z_{(1-\alpha/2)} \approx$
90%	1.64
95%	1.96
99%	2.58

MegaStat Application

Example 6.5.1.

- A report published in 2003 reported that 18 percent of Internet users have used it to search for information regarding experimental treatments or medicines. The sample consisted of 1220 adult Internet users. Construct a 95% confidence interval for the proportion of Internet users in the sampled population who have searched for information on experimental treatments or medicines.

Example 6.5.1.

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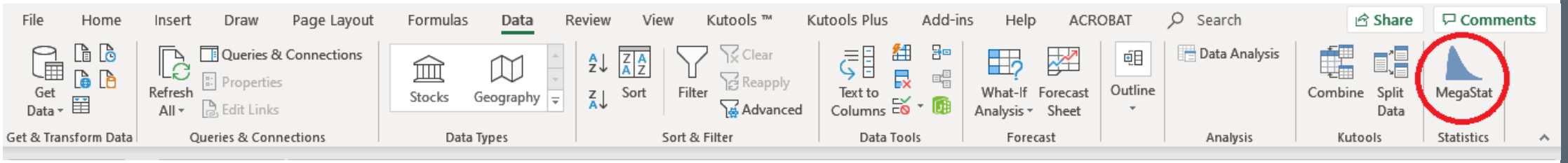
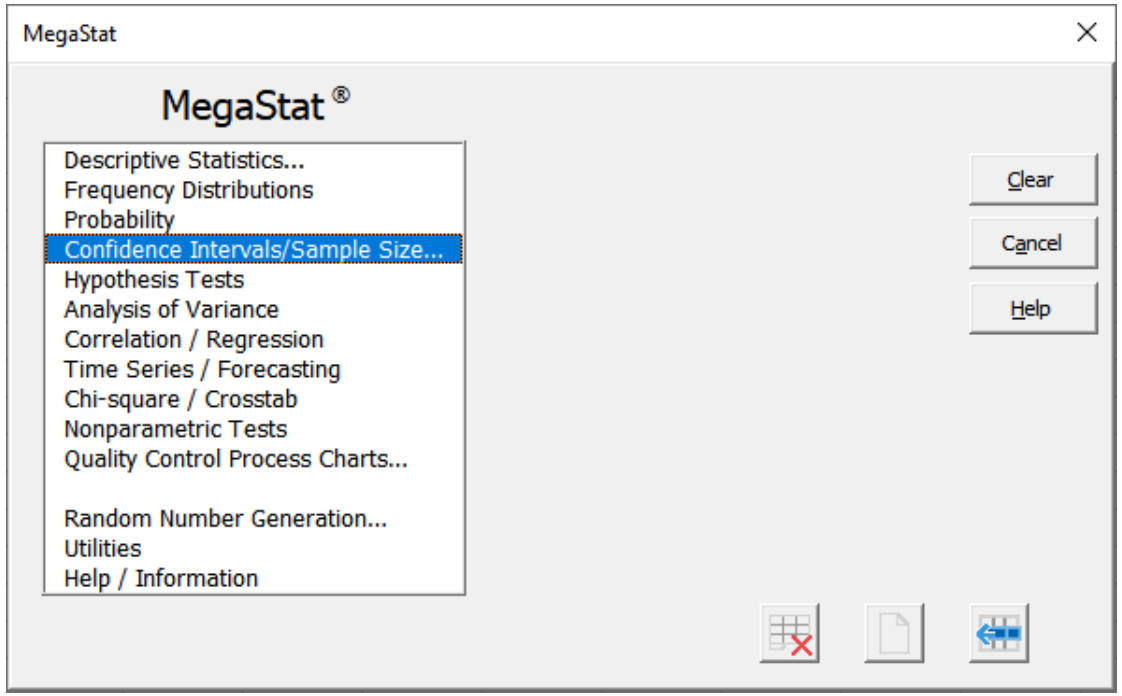
Interval Estimation for p (MegaStat Application)





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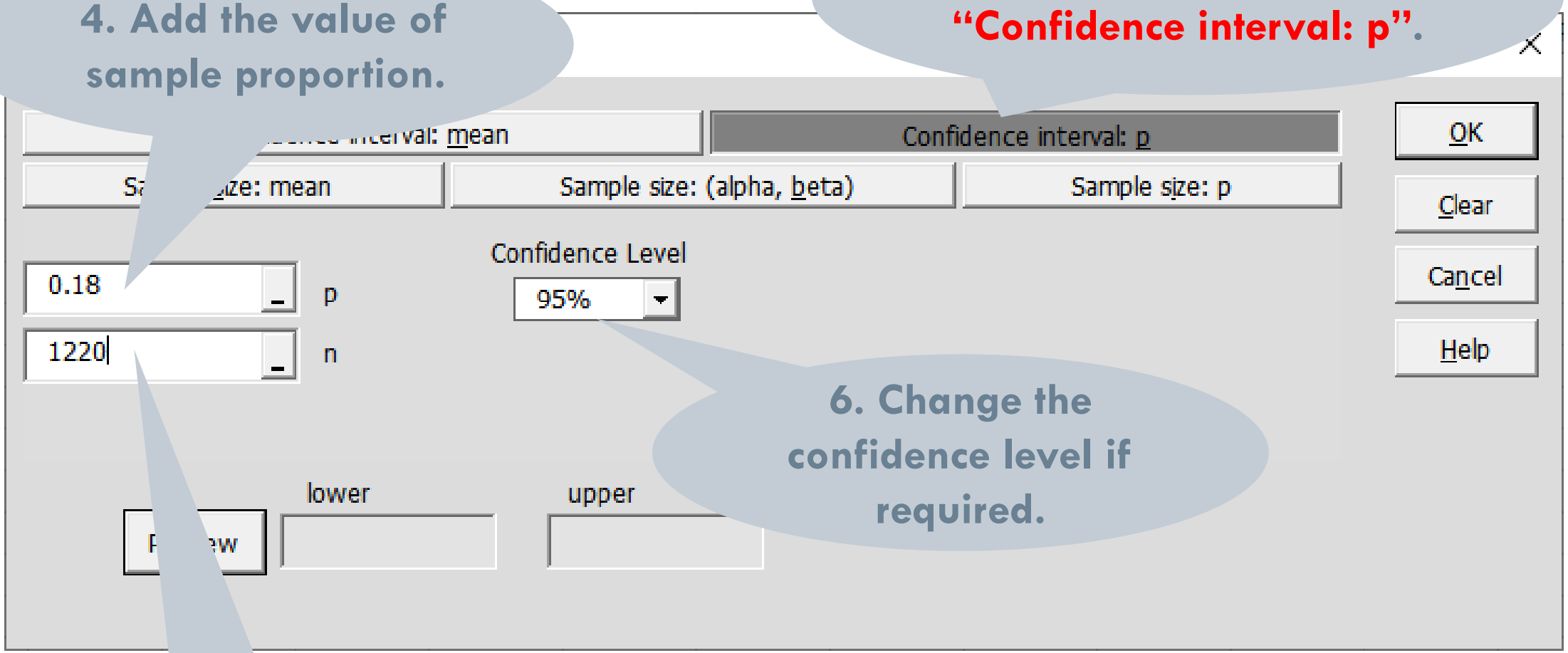
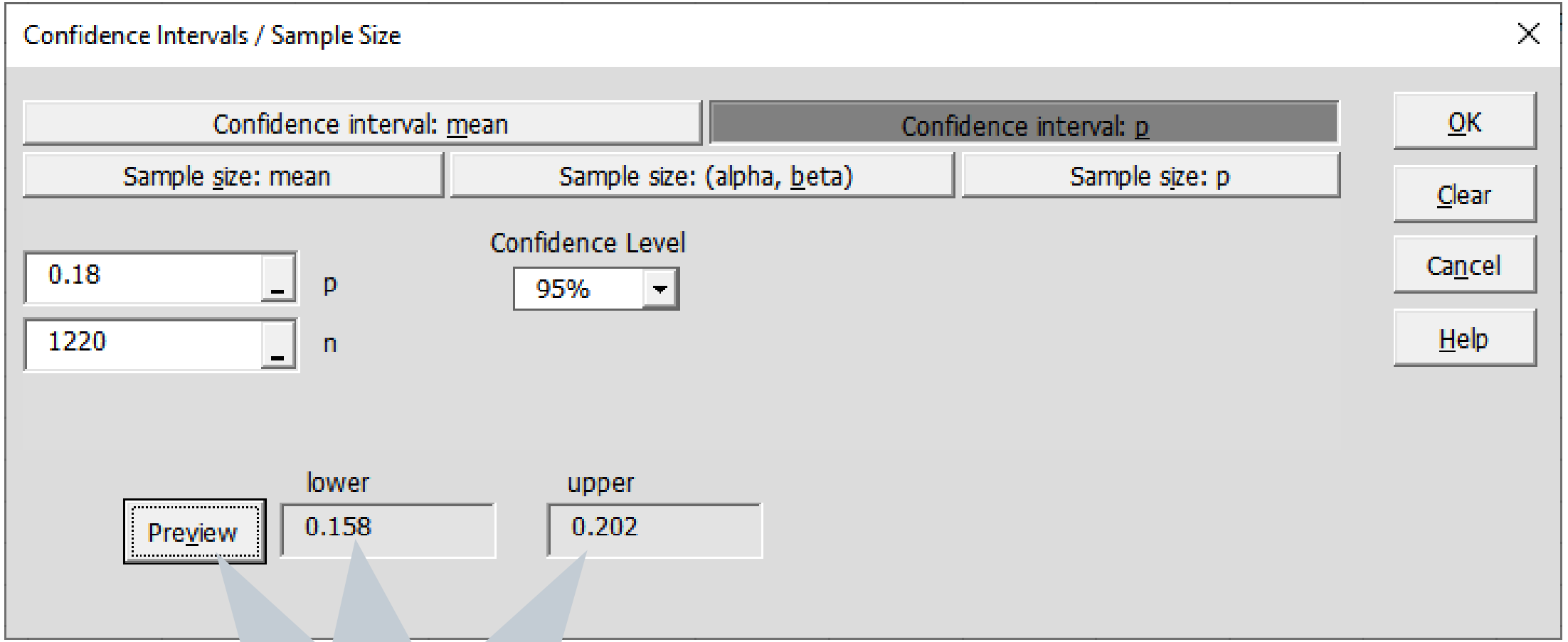

4. Add the value of sample proportion.
3. Choose the tab "Confidence interval: p".
Confidence interval: mean
Confidence interval: p
Sample size: mean
Sample size: (alpha, beta)
Sample size: p
OK
Clear
Cancel
Help
Confidence Level
0.18
p
95%
1220
n
6. Change the confidence level if required.
lower
upper
5. Add the value of the sample size.



7. Click Preview to get the confidence interval limits.